

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY GURAJADA VIZIANAGARAM
I B. Tech II Semester Supplementary Examinations January-2025
Differential Equations and Vector Calculus

(Common to all branches)

Time: 3 hours

Max. Marks: 70

Question paper consists of Part A & Part B.
Part A is compulsory, Answer all questions.
In Part B, Answer any one question from each unit.

PART-A**(20 Marks)**

- 1 a) Find the order and degree of the D.E $\frac{dy}{dx} + xy = 0$ [2]
- b) Write the RL circuit and define the terms in the circuit [2]
- c) Solve the D.E $(D^2 + 4D + 4)y = 0$ [2]
- d) Find the P.I of $(D^2 - 4D + 4)y = e^{2x}$ [2]
- e) Verify whether $\left(\frac{\partial^2 z}{\partial x^2}\right)^3 - \frac{\partial^2 z}{\partial y^2} = e^x \sin y$ is linear PDE [2]
- f) Solve the PDE $(D^2 - a^2 D^2)z = 0$ [2]
- g) Find grad ϕ where $\phi = x^3 + y^3 + 3xyz$ [2]
- h) Find the maximum value of the directional derivative of $\phi = 2x^2 - y - z^4$ at $(2, -1, 1)$ [2]
- i) Define surface integral [2]
- j) State stoke's theorem [2]

PART-B**(50 Marks)****Unit-1**

- 2 a) Solve the D.E $(x^2 y^2 + xy + 1)ydx + (x^2 y^2 - xy + 1)xdy = 0$ [5]
- b) If the air is maintained at 30°C and the water at the temperature 100°C cools down to 80°C in 10 min. Then find when the temperature of water will become 40°C [5]

(OR)

- 3 a) Solve the D.E $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ [5]
- b) Solve the D.E $(3x^2 - y^3)dx - (2x^2 y - xy^2)dy = 0$ [5]

Unit-2

- 4 Solve the D.E $(D^2 + 4)y = x \sin 2x$ [10]

(OR)

- 5 a) Solve the D.E $(D^2 + 2D + 1)y = x^2 + 2x + 1$ [5]
- b) Solve the D.E $(D^2 + 5D + 6)y = \sin 2x$ [5]

Unit-3

- 6 a) Form the PDE $z = (\sqrt{x+a})(\sqrt{y+b})$ by eliminating the constants a & b [5]
- b) Solve the PDE $xp - yq = y^2 - x^2$ [5]

(OR)

- 7 a) Form the PDE by eliminating arbitrary function from $z = f(2x + 3y) + g(3x - y)$ [5]
b) Solve the PDE $(y - zx)p + (x + yz)q = x^2 + y^2$ [5]

Unit-4

- 8 a) Find the directional derivative of $\phi = xyz$ in the directional of $\vec{i} + \vec{j} + \vec{k}$ at $(1, 2, 3)$ [5]
b) Verify whether $(y + z)\vec{i} + (x + z)\vec{j} + (y + x)\vec{k}$ is solenoidal [5]

(OR)

- 9 a) show that the vector $\vec{f} = (\log y + z)\vec{i} + \left(\frac{x}{y} - z\right)\vec{j} + (x - y)\vec{k}$ is irrotational [5]
b) Prove that $\text{curl grad } \phi = 0$ [5]

Unit-5

- 10 a) If $\vec{F} = (x - y)\vec{i} + (x + y)\vec{j}$ Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ around the curve $y = x^2$ and $y^2 = x$ [5]
b) Evaluate $\oint_C (x^2 + y^2)dx + 3xy^2 dy$ where c is the circle $x^2 + y^2 = 4$ in xy plane using Green's theorem [5]

(OR)

- 11 Verify Gauss Divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped $0 \leq x \leq a ; 0 \leq y \leq b ; 0 \leq z \leq c$. [10]
